2019 Academic Year
Department of Management Science and Technology
Graduate School of Engineering
Tohoku University

[Special Selection Program for Foreign Students]

Mathematics

March 4, 2019

9：30〜11：30 (120 minutes)

Directions

1. Do not open these papers without permission.
2. <u>Enter your examinee number</u> in all answer sheets.
3. <u>Solve all four problems.</u>
4. <u>Use one answer sheet for one problem.</u> If it is not possible to write on one side, you may use the other side. Do not use more than one answer sheet for one problem.
5. After submitting your answer sheets, you need to keep sitting down.
6. Since examination papers must be collected, <u>do not bring them home.</u>

Problem 1

(1) Plot the solid figure defined by the following function, and find the volume and the centroid.

$$x^2 + y^2 - (z - \mathrm{a})^2 = 0, \qquad 0 \le z \le \mathrm{a} \quad (\mathrm{a} : \text{positive constant})$$

(2) Answer the following questions.

$$y\frac{d^2 y}{dx^2} + y\frac{dy}{dx} - 2\left(\frac{dy}{dx}\right)^2 = 0 \qquad \text{(Eq. 1)}$$

(a) Let $s = \dfrac{dy}{dx}$, obtain the first-order linear differential equation with s and y by rearranging Eq.1 when $s \neq 0$.

(b) Solve Eq. 1.

Problem 2

In an orthogonal coordinate system, when $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ are unit vectors in the x, y, z directions, respectively, answer the following questions.

(1) If $\boldsymbol{F} = 2xy\boldsymbol{i} - y^2\boldsymbol{j}$, evalutate

$$\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$$

where C is the curve in the xy plane, $y = 3x^2$, from $(0, 0)$ to $(1, 3)$.

(2) Find a unit normal to the surface $x^2y - 2xz = 8$ at the point $(1, 2, -3)$.

(3) If $\varphi(x, y, z) = xy^2z$ and $\boldsymbol{A} = xz\boldsymbol{i} - xy^2\boldsymbol{j} + \mathrm{y}\mathrm{z}^2\boldsymbol{k}$, find

$$\frac{\partial^3}{\partial x^2 \partial z}(\varphi \boldsymbol{A})$$

at the point $(2, -1, 1)$.

Problem 3

Suppose a sequence $x_1, x_2, x_3, \cdots$ that satisfies:

$$x_1 = x_2 = 1$$

$$x_n = 2x_{n-1} + x_{n-2} \quad (n \geq 3)$$

Let $\boldsymbol{X}_n = (x_n, x_{n-1})$. Then $\boldsymbol{X}_n$ follows the following recurrence formula:

$$\boldsymbol{X}_n = \boldsymbol{A}\boldsymbol{X}_{n-1} \quad (n \geq 3)$$

where $\boldsymbol{A}$ is a 2x2 square matrix. Answer the following questions from (1) to (5).

(1) Obtain x_3, x_4 and x_5.

(2) Write down the matrix $\boldsymbol{A}$.

(3) Find the eigenvalues and eigenvectors of $\boldsymbol{A}$.

(4) Diagonalize $\boldsymbol{A}$.

(5) Express the nth element $x_n (n \geq 3)$ as a function of n.

Problem 4

Five 6th-grade boys were randomly selected, and sprint times of 50m were recorded for each boy wearing two types of shoes. The recorded results are shown in the table below. Suppose the sprint conditions are the same except for the types of shoes, and the 50m sprint times of 6th-grade boys follow the normal distribution. Solve the following problems. Here, the critical value of t-distribution (two-sided) at a significance level of 5% with 4 degrees of freedom is 2.78.

	Boy 1	Boy 2	Boy 3	Boy 4	Boy 5
Sprint time with the shoes type A [s]	9.0	8.2	9.1	9.2	8.5
Sprint time with the shoes type B [s]	8.7	8.4	8.7	8.8	8.9

（1）What are the average and variation of the sprint times of these five 6th-grade boys? Find the values for each case of wearing the shoes type A and B.

（2）What are the average and variation of the difference of sprint times of each boy when wearing the two types of shoes.

（3）Prove that the 50m sprint times of the five boys with the two types of shoes are not independent.

（4）Suppose a null hypothesis that the 50m sprint times of the five boys with the two types of shoes are different, conduct a hypothesis test at a significance level of 5%.