

2017 Academic Year
Department of Management Science and Technology
Graduate School of Engineering
Tohoku University
[Special Selection Program for Foreign Students]

Mathematics

1 March 2017

9:30-11:30 (120 minutes)

Instructions

1. Do not open question sheets until you are allowed to start answering.
2. **Fill out examinee's number in each answer sheet.**
3. After submitting answer sheets, you must be seated until you are allowed to leave the room.
4. **Do not take question sheets home.** They must be collected.
5. Write clearly. Unreadable answers will not be considered.
6. Solve all four problems.
7. Use one answer sheet for one problem. If it is not possible to write on one side, you may use the other side.

Problem 1

- (1) Find a general solution to the following differential equation.

$$\frac{dy}{dx} = -y + \cos x$$

- (2) Find the power series solution of the following differential equation around $x = 0$.

$$\frac{dy}{dx} = 3y + 2$$

- (3) Find the following integral.

$$\int_0^1 \frac{1}{(x-3)^2(x-2)} dx$$

Problem 2

- (1) Find an equation for the plane determined by the points $P_1(-1, 3, 1)$, $P_2(2, -1, 2)$, $P_3(3, 2, 1)$.
- (2) Find the projection of the vector $\mathbf{A} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ on the vector $\mathbf{B} = 7\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}$.
- (3) Find $\cos\theta$ where θ is the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.
- (4) If the vector field \mathbf{F} is given by $\mathbf{F} = 3xy\mathbf{i} - y^2\mathbf{j}$, evaluate

$$\int_C \mathbf{F} \, d\mathbf{r}$$

where C is the curve in the x - y plane, $y = 2x^2$, from $(0, 0)$ to $(1, 2)$.

Problem 3

Let \mathbf{A} be the following square matrix:

$$\mathbf{A} = \begin{bmatrix} a & b/2 & 0 \\ 1-a & a & b \\ 0 & (1-a)/2 & a \end{bmatrix},$$

where a and b are real numbers and $0 < a < 1$.

- (1) Find all eigenvalues of \mathbf{A} .
- (2) Describe the necessary and sufficient conditions for the largest eigenvalue of \mathbf{A} to be 1.
- (3) Suppose that a and b satisfy the above condition. Diagonalize \mathbf{A} .

Problem 4

A company produces a coffee bag product, and declares that the net weight of a coffee bag is 90 g. For this company's coffee bag, it is known that the standard deviation of net weight is 4 g. Nine samples of coffee bags of this company were randomly selected to check the net weight of each, and the following result was obtained.

85, 93, 94, 86, 97, 96, 94, 92, 91 g

Solve the following problems.

Here, the standard normal distribution with a mean of 0 and a variation of 1 is expressed by

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right),$$

and the cumulative distribution function $F(x)$ of $f(x)$ holds

$$F(1.96) = \int_{-\infty}^{1.96} f(x) dx = 0.9750.$$

- (1) What is the mean net weight of the nine samples?
- (2) What is the variation of net weight of the nine samples?

Suppose the null hypothesis H_0 claims that net weight of coffee bags follows a normal distribution and the mean net weight and the standard deviation are 90 g and 4 g, respectively, and the alternative hypothesis H_1 claims that the mean net weight is not 90 g.

- (3) If H_0 holds, what is the standard deviation of the sampling distribution of mean net weight of nine coffee bags?
- (4) Conduct a hypothesis test at a significance level of 5%.