2012 Entrance Examination
Department of Management Science and Technology
Graduate School of Engineering
Tohoku University

Mathematics

February 28, 2012

9：30～11：30 (120 minutes)

Directions

1. Do not open these papers without permission.
2. Enter your examinee number in all answer sheets.
3. Solve four problems out of five problems.
4. Use one answer sheet for one problem. If it is not possible to write on one side, you may use the other side. Do not use more than one answer sheet for one problem.
5. After submitting your answer sheets, you need to keep sitting down.
6. Since examination papers must be collected, do not bring home.

1

(1) Solve the following differential equation:

$$2x^2 y\frac{dy}{dx} + x(1+y^2) = 0$$

(2) Solve the following simultaneous differential equation:

$$\begin{cases} \dfrac{dy_1}{dx} = y_1 + y_2 - y_3 \\ \dfrac{dy_2}{dx} = 2y_2 \\ \dfrac{dy_3}{dx} = y_2 - y_3 \end{cases}$$

2

Let W be the three dimensional region bounded by $z = 0$, $z = \exp(x^2 + y^2)$, $x^2 + y^2 = 1$, $x^2 + y^2 = 2$. Let $\boldsymbol{F} = (4x - xy)\boldsymbol{i} - y\boldsymbol{j} + yz\boldsymbol{k}$ be vector field, where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are unit vectors in x, y, and z directions, respectively.

(1) Find the volume of W.

(2) Find div$\boldsymbol{F}$.

(3) Find the flux of the vector field $\boldsymbol{F}$ out of the region W.

3

(1) Find the equation of the tangent plane to the surface $z = x^2 + y^2$ at the point $(a, b, a^2 + b^2)$.

(2) Find the volume V of the solid bounded by $z = x^2 + y^2$, $(x-1)^2 + y^2 = 1$ and the tangent plane found in (1). The solid is inside the cylinder surface $(x-1)^2 + y^2 = 1$.

(3) Find the minimum value of V.

4

(1) Find eigenvalues and normalized eigenvectors for matrix A.

$$A = \begin{pmatrix} 3 & 1 & -3 \\ -7 & -2 & 9 \\ -2 & -1 & 4 \end{pmatrix}$$

(2) Find A^n.

5

Answer the following questions.

(1) Let $X_1, X_2, \cdots, X_n$ be statistically independent random variables with identical probability distribution of average μ and variance σ^2. We define a new random variable S^2 as follows:

$$S^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2 ,$$

where $$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i .$$

Find the expressions of expectation $E\left(S^2\right)$ with μ and σ^2.

(2) Find the probability density functions g(y) of random variables Y expressed by the following functions when X is the random variable with uniform probability density function U(1,0).

(a) $Y = X^2 + 2$

(b) $Y = \cos\left(\frac{\pi}{2} X\right)$,

where,

$$U(0,1) = \begin{cases} 0 & (x \le 0) \\ 1 & (0 < x < 1) \\ 0 & (x \ge 1) \end{cases}$$