2010 Entrance Examination
Department of Management Science and Technology
Graduate School of Engineering
Tohoku University

Mathematics

February 28, 2011

9:30～11:30 (120 minutes)

Directions

1. Do not open these papers without permission.
2. Enter your examinee number in all answer sheets.
3. Solve four problems out of five problems.
4. Use one answer sheet for one problem. If it is not possible to write on one side, you may use the other side. Do not use more than one answer sheet for one problem.
5. After submitting your answer sheets, you need to keep sitting down.
6. Since examination papers must be collected, do not bring home.

1

(1) Solve the following differential equation:

$$\frac{dy}{dx} = \frac{x - y}{x + y}$$

(2) Solve the following simultaneous differential equation:

$$\begin{cases} \dfrac{dy_1}{dx} + 6y_1 + \dfrac{dy_2}{dx} + 3y_2 = 0 \\ \dfrac{dy_1}{dx} - \dfrac{dy_2}{dx} + y_2 = 0 \end{cases}$$

2

Let $\boldsymbol{A} = (y - 2xz)\boldsymbol{i} - (3x + z)\boldsymbol{j} + y\boldsymbol{k}$ and $\boldsymbol{B} = -(2y + xz)\boldsymbol{i} + 2z\boldsymbol{j} - (x^2 - y)\boldsymbol{k}$ be vector fields, and let C be the line segment from (0, 0, 0) to (1, 1, 2), where, $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are unit vectors in x, y, and z directions, respectively.

(1) Find the line integral of the vector field $\boldsymbol{A}$ along C.

(2) Find the constant 'a' so that the vector field $\boldsymbol{U} = \boldsymbol{A} + a\boldsymbol{B}$ has a scalar potential φ given by $\boldsymbol{U} = \mathrm{grad}\varphi$. Then find the scalar potential φ and the line integral of the vector field $\boldsymbol{U}$ along C.

3

(1) Suppose we want to approximate the function $y = e^x$ by a linear function $y = ax + b$ over the range $0 \le x \le 1$. Find the values a and b that minimize the error $E(a, b)$ given by

$$E(a, b) = \int_0^1 \{e^x - (ax + b)\}^2 dx.$$

(2) Let D be a plane region given by the simultaneous equations $x^2 + \mathrm{y}^2 \le 2$, $(x - 1)^2 + \mathrm{y}^2 \ge 1$. Find the area of D.

4

(1) Find eigenvalues and normalized eigenvectors for matrix *A*.

$$A = \begin{pmatrix} 4 & -2 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

(2) Find A^n.

(3) Assuming A^0 is identity matrix, find $\sum_{n=0}^{\infty} \frac{A^n}{n!}$.

5

(1) The boxes S_i and T_i $(i=1,2,\cdots,m)$ in the electric circuit A and B of Fig.1 and Fig.2 represent fuses. When electric currents flow on those fuses, they cut with the probabilities p_S *and* p_T denoted in the boxes. Find the probabilities p_A and p_B that electric currents flow on the circuit A and B.

(2) When the number of fuse boxes m is equal to two in the circuit A and B of Fig.1 and Fig.2, find the probabilities p_A and p_B , and compare the magnitudes of p_A and p_B .

(3) Let X be a random variable with the following probability density function $f_X(x)$. Find the probability density function $f_Y(y)$ of the random variable $Y = 2X + 3$.

$$f_X(x) = \begin{cases} 1/2 & (-1 \leqq x \leqq 1) \\ 0 & otherwise \end{cases}$$

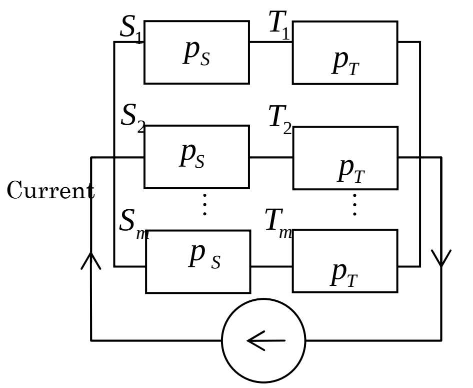

Fig.1 circuit A

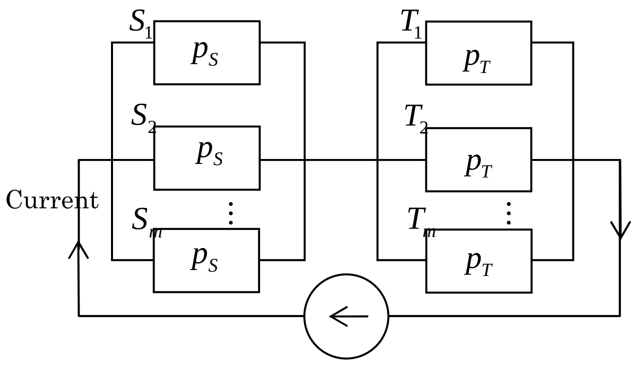

Fig.2 circuit B