2010 Entrance Examination
Department of Management Science and Technology
Graduate School of Engineering
Tohoku University

Mathematics

August 23, 2010

9：30～11：30 (120 minutes)

Directions

1. Do not open these papers without permission.
2. Enter your examinee number in all answer sheets.
3. Solve four problems out of five problems.
4. Use one answer sheet for one problem. If it is not possible to write on one side, you may use the other side. Do not use more than one answer sheet for one problem.
5. After submitting your answer sheets, you need to keep sitting down.
6. Since examination papers must be collected, do not bring home.

1

(1) Solve the following differential equation:

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$$

(2) Solve the following simultaneous differential equation:

$$\begin{cases} \dfrac{dx_1}{dt} = x_1 - x_2 + x_3 \\ \dfrac{dx_2}{dt} = x_1 + x_2 - x_3 \\ \dfrac{dx_3}{dt} = 2x_1 - x_2 \end{cases}$$

2

Consider the vector field $\boldsymbol{A} = zx\boldsymbol{i} + \frac{y^2}{x}\boldsymbol{j} + x^2\boldsymbol{k}$ and the surface S given by $x^2 + y^2 + z^2 = 9$ for $z \geq 0$, where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are unit vectors in x, y and z directions, respectively.

(1) Show that there is no scalar potential φ such that $\boldsymbol{A} = \mathrm{grad}\varphi$.

(2) Find the unit normal vector to the surface S at the point (2, 1, 2).

(3) Verify Stokes' theorem

$$\oint_{\mathrm{C}} \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{r} = \iint_{\mathrm{S}} rot\boldsymbol{A} \cdot \boldsymbol{n}\, \mathrm{dS}\,,$$

where C is the intersection of the xy-plane with the surface S.

3

(1) A function $f(x)$ is differentiable for all x, and verifies following equations. Determine $f(x)$.

$$f(x) = f(-x) + 2x$$

$$f(x)f'(x) + f(-x)f'(-x) = 6x^2 + 2$$

(2) Consider 2 points $P(x, y, z)$ and $Q(x, y, 0)$, where x, y, z are functions of time t:

$$x = e^{2t} \cos t,\ y = e^{2t} \sin t,\ z = t + 1$$

a) Find the length of curve traced by the point Q from $t = 0$ to $t = a$ $(a > 0)$.

b) Find the surface of area depicted by the segment PQ from $t = 0$ to $t = a$ $(a > 0)$.

4

（1） $For\ A = \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix},\ find\ A^n.$

（2） Consider a personal computer market where consumers purchase either computer A or computer B every period. The number of consumers is constant. a_n is defined as the number of computers A sold in the period of n divided by the total number of computers sold in the period of n. b_n is defined as the number of computers B sold in the period of n divided by the total number of computers sold in the period of n $(a_n + b_n = 1)$. Seventy percent of consumers who purchased A in the $n-1$ period purchase A in the n period. Thirty percent of consumers who purchased A in the $n-1$ period purchase B in the n period. Eighty percent of consumers who purchased B in the $n-1$ period purchase B in the n period. Twenty percent of consumers who purchased B in the $n-1$ period purchase A in the n period. Define the relationship between (a_n, b_n) and (a_{n-1}, b_{n-1}) using a matrix. Furthermore, assuming that n is sufficiently large, find a_n / b_n.

5

(1) The boxes A and B in an electric circuit of Fig.1 represent fuses. When electric currents flow on those fuses, they cut with the probabilities p_A, p_B denoted in the boxes. Find the probability p_1 that electric current flows on the circuit.

(2) Find the probability p_2 that electric current flows on a circuit in Fig.2. Each box represents fuse. The parameters $p_A, p_B, \cdots, p_E$ denoted in the boxes are the probabilities that the fuses cut when electric currents flow on the fuses.

(3) Let $\mathrm{X}_1, \mathrm{X}_2, \cdots, \mathrm{X}_\mathrm{n}$ be statistically independent random variables with identical probability distribution of average μ and variance σ^2. We define a new random variable $\overline{\mathrm{X}}$ as follows:

$$\overline{\mathrm{X}} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{X}_\mathrm{i}$$

Find the expressions of expectation $\mathrm{E}(\overline{\mathrm{X}})$ and variance $\mathrm{V}(\overline{\mathrm{X}})$ with μ and σ^2.

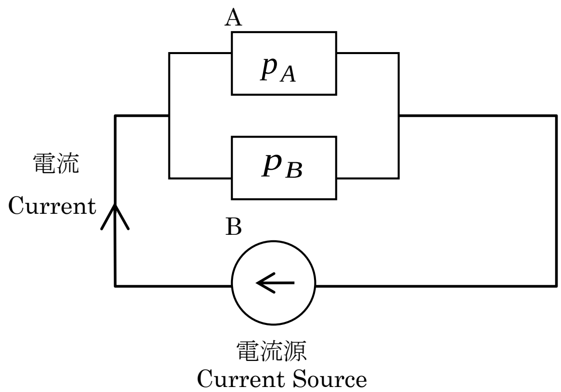

図 1
Fig.1

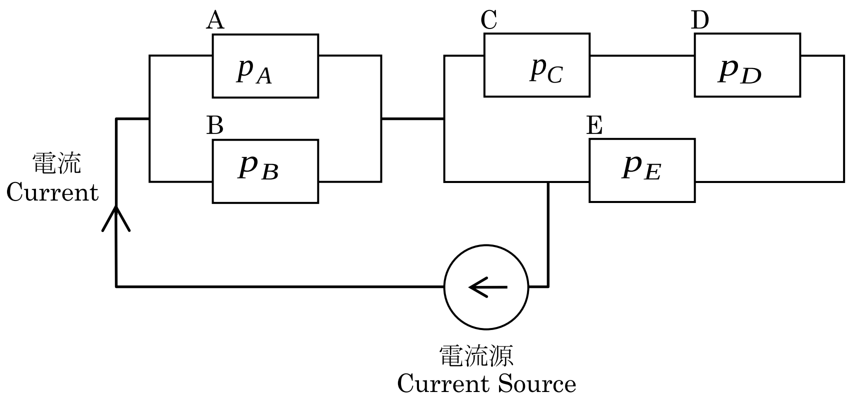

図 2
Fig.2