2007 Entrance Examination
Department of Management Science and Technology
Graduate School of Engineering
Tohoku University

Mathematics

August 29, 2007

Schedule and Time: 9：30～11：30 (120 minutes)

Directions

1. Do not open these papers without permission.
2. Enter your examinee number in all answer sheets.
3. Solve four problems out of five problems.
4. Use one answer sheet for one problem. If it is not possible to write on one side, you may use the other side. Do not use more than one answer sheet for one problem.
5. After submitting your answer sheets, you need to keep sitting down.
6. Since examination papers must be collected, do not bring home.

1. Answer the following questions.

(1) Find the volume of the ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{3a^2}+\dfrac{z^2}{3a^2}=1$.

(2) Find the tangential plane to the ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{3a^2}+\dfrac{z^2}{3a^2}=1$ at (x_1, y_1, z_1) on the surface of the ellipsoid where $x_1 \neq 0,\ y_1 \neq 0$ and $z_1 \neq 0$. Find the normal to the tangential plane, too.

2. Answer the following questions for the vector field

$$F(x,y)=\left(p(x,y),q(x,y)\right)=\left(\frac{-y}{x^2+y^2},\frac{x}{x^2+y^2}\right)$$

(1) Find the curvilinear integral of $F(x,y)$ from point $(0,1)$ to $(1,0)$ along the line C_1 as shown in Fig. 1 (a)

(2) Find the curvilinear integral of $F(x,y)$ along the directed closed contour C as shown in Fig. 1 (b). The direction of the contour C is counterclockwise. If necessary, you can use the Green theorem

$$\int_C (pdx+qdy)=\iint_A\left(\frac{\partial q}{\partial x}-\frac{\partial p}{\partial y}\right)dxdy,$$

where, A denotes the area within the directed closed contour C and the direction of the contour C is defined such that the area A exists on the left side of the contour.

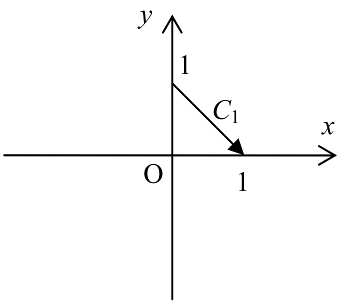

Fig. 1 (a)

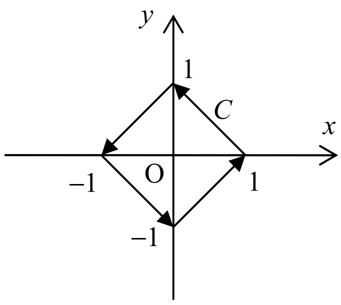

Fig. 1 (b)

3. Answer the following questions.

(1) Obtain general solutions of the following simultaneous differential equation.

$$\frac{dx}{dz} = x - 2y$$
$$\frac{dy}{dz} = -3x + 2y$$

(2) As shown in Fig. 2, a metallic stick of L in length generates heat by constant calorie q_0 for each unit length. Obtain the expression that shows the temperature distribution in the metallic stick when both ends of the metallic stick are fixed to constant temperature T_0. Moreover, show the appearance of the expression in the figure. The equation of heat conduction of the steady state that expresses the relation between generated heat $q(x)$ and temperature $T(x)$ at x in the stick is assumed to be given as follows:

$$a\frac{d^2T(x)}{dx^2} + q(x) = 0$$

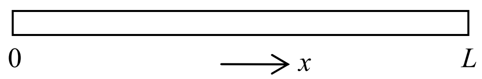

Fig. 2

4. For the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & 2 & 3 \end{bmatrix},$$

answer the following questions.

(1) Find all the eigenvalues and eigenvectors of the matrix A.

(2) Find the matrix S and the diagonal matrix Λ that satisfy $A = S\Lambda S^{-1}$.

(3) Calculate the sum of the infinite series:

$$\sum_{i=0}^{\infty} \frac{A^i}{i!},$$

where A^0 is the identity matrix.

5. The probability density function of the random variable X is denoted by

$$f(x) = \begin{cases} -a(x^2-1) & \text{for } -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases},$$

where a is a positive constant. Answer the following questions.

(1) Find the value of a and the cumulative distribution function of X.

(2) Find the mean μ and the variance σ^2 of X using the value of a obtained in (1).

(3) The kurtosis of X is defined as

$$k = \frac{E\left((X-\mu)^4\right)}{\sigma^4}$$

where $E(\ )$ is the expected value. Find the kurtosis k of X.